The Mathematics Educator
1997, Vol. 2, No. 1, 93-112

Difficulties With Problem Solving In Mathematics

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Abstract

This review of the research literature on difficulties with problem solving in mathematics shows us that problem solving in mathematics is a complex process which requires an individual who is engaged in a mathematical task to coordinate and manage domain-specific and domain-general pieces of knowledge. It also suggests that (i) the mathematics content level of the problems which students at different year levels of schooling will be able to solve successfully and (ii) the different strategies or heuristics which students at different year levels use to solve the same mathematical problems must govern the design of problem-solving curricula at the various year levels of schooling.

The Nature of Mathematical Problem Solving

In a historical review focussing on the role of problem solving in the mathematics curriculum, Stanic and Kilpatrick (1989) wrote:

> *Problems have occupied a central place in the school mathematics curriculum since antiquity but problem solving has not. (p. 1)*

A common view among mathematics teachers, students and parents is that, "Doing mathematics is solving problems" and "Mathematics is about how to solve problems".

In a position paper on basic skills the National Council of Supervisors of Mathematics (1977) stated that:

> *Learning to solve problems is the principal reason for studying mathematics. (p. 20)*

Cockcroft (1982) also attempted to characterise problem solving:

> *The ability to solve problems is at the heart of mathematics. Mathematics is only useful to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name 'problem solving'. (para 249)*

From the literature it appears that some writers believe that solving problems is the essense of mathematics learning, while others consider mathematics as a body of knowledge which provides the tools for the process of solving mathematical problems.

Prior to the 1980's, before "problem solving" became the focus of much mathematics education research, it tended to be subsumed under the label "mathematical thinking" in the area of cognitive psychology of mathematics. Burton (1984) made a clear distinction between mathematical thinking and the body of knowledge described as mathematics. She emphasised that mathematical thinking is not thinking about the subject matter (mathematics) but a way of thinking which relies on mathematical operations. Mathematical problems are the starting points of mathematical inquiry which lead to thinking. Law (1972) contended that thinking takes place when a person meets a problem and accepts the mental challenge it offers and Burton (1984) added that:

> *If thinking is a way of improving understanding and extending control over the environment, mathematical thinking uses particular means to do this, means that can be recognized as arising from or pertaining to the study of mathematics. (p. 36)*

But what then is a problem in mathematics? Krulik and Rudnick (1988) defined a problem as "a situation . . . that requires resolution and for which the individual sees no apparent or obvious means or path to obtaining the solution" (p. 3). Schoenfeld (1989) stated that:

> *... for any student a mathematical problem is a task*
>
> *a) in which the student is interested and engaged and for which he/she wishes to obtain a resolution, and*
>
> *b) for which the student does not have a readily accessible mathematical means by which to achieve that resolution. (pp. 87-88)*

Owing to differences in knowledge, experience, ability or interest, a problem for one person may not be a problem for another. Also a problem for someone at a particular time may not be so at another time. In some contexts, as students develop their mathematical ability, what were problems initially may after some practice become mere exercises.

It follows that mathematical problem solving is a complex process which requires an individual to coordinate previous experiences, mathematical knowledge, understanding and intuition, in order to satisfy the demands of a novel situation. Garofalo and Lester (1985) claimed that problem solving has come to be viewed as a process involving the highest faculties - visualisation, association, abstraction, comprehension, manipulation, reasoning, analysis, synthesis, generalisation - each needing to be "managed" and all needing to be "coordinated".

The process of establishing relationships and making connections between concepts associated with mathematical content (topics) in a novel situation is one of the most important aspects of problem-solving activities. An artificial separation of the process from the content in the classroom instructional programme was cautioned by Lesh (1981). He maintained that students do not first learn the mathematics, then learn to solve problems using the mathematics and finally learn to solve applied problems. There is a dynamic interaction between basic mathematical concepts and facts, and important applied problem solving processes.

Classifying Types of Problem Solvers

In the literature one finds references to "good" and "poor", "expert" and "novice", "successful" and "unsuccessful" problem solvers among many other categories. Comparing the behaviours between successful and unsuccessful problem solvers, Dodson (1972) found that good problem solvers were superior with respect to:

a) overall mathematics achievement,
b) verbal and general reasoning ability,
c) spatial ability,
d) positive attitudes,
e) resistance to distraction,
f) level of field independence, and
g) divergent thinking.

Krutetskii (1976) found that a major difference between good and poor problem solvers lay in their perception of the important elements of the problems they were attempting to solve. Good problem solvers typically had certain abilities that poorer problem solvers lacked:

a) the ability to distinguish relevant from irrelevant information,
b) the ability to see quickly and accurately the mathematical structure of a problem,
c) the ability to generalise across a wide range of similar problems, and
d) the ability to remember a problem's formal structure for a long time.

Krutetskii also noted that good problem solvers tended to recall the structural characteristics of the problem and to forget its details, whereas poor problem solvers tended to recall specific details of the problem. Silver (1979) observed this same difference between good and poor problem solvers in their recall of problem structure, however, unlike Krutetskii, good problem solvers in Silver's study tended to have reasonably accurate recall of contextual details of the problems. Silver indicated that the poor problem solvers' difficulties in recalling the structure of the problems could be attributed to their lack of ability in noticing the problem structure, rather than a general lack of memory, since poor problem solvers exhibited more accurate recall of problem details than did most good problem solvers.

Schoenfeld's (1985, 1987) research suggested that good problem solvers can be distinguished from poor problem solvers in at least five important ways:

1. The knowledge of good problem solvers is well connected and composed of rich schemata while that of poor problem solvers is not.

2. Good problem solvers tend to focus their attention on structural features of problems while poor problem solvers focus on surface features.

3. Good problem solvers are more aware than poor problem solvers of their strengths and weaknesses as problem solvers.

4. Good problem solvers are better than poor problem solvers at monitoring and regulating their problem-solving efforts.

5. Good problem solvers tend to be more concerned than poor problem solvers about obtaining "elegant" solutions to problems.

Heller and Hungate (1985) reviewed several empirical and theoretical analyses related to scientific problem solving and noted that novices tend to be quite deficient with respect to understanding or perceiving problems in terms of fundamental principles or concepts. They cannot, or do not, construct problem representations that are helpful in achieving solutions. Experts solve problems using a process of successive refinements - unless they are faced with a simple problem for which they can immediately recall a specific solution method. The strategy used by experts is to perform high-level planning and qualitative analysis before beginning to generate equations. Novices do not have the knowledge required to approach problems in this way, and tend to go directly from the problem text to equations. Experts have a large amount of domain-specific factual knowledge that is both technically correct and well organised. Experts also have knowledge about when concepts and principles are applicable and useful, and procedures for interpreting and applying their factual knowledge. Novices lack in much of this knowledge, do not have their knowledge well organised, and frequently exhibit naive preconceptions rather than scientifically correct ideas. Experts have repertoires of familiar patterns and knowledge of problem types and solution methods which novices have not yet developed.

Foong (1990, 1994) in her studies of pre-service teachers who were training to be mathematics teachers found that:

1. Successful problem solvers translated the problem statement more correctly and more exactly than did unsuccessful problem solvers.

2. Unsuccessful problem solvers tended to attend to obvious details, translating statement by statement without having a global representation of the problem situation.

3. Successful problem solvers planned their solutions in more detail before carrying them out than unsuccessful solvers, who tended to be impulsive in executing a solution without a complete understanding of the problem.

4. Unsuccessful problem solvers tended towards impulsive solutions and when in difficulty they often returned to the same incorrect method, sometimes repeatedly.

5. Successful problem solvers used more metacognitive processes which were task directed, showing greater awareness of how things were in the solution path and where they should be going in the process.

6. Negative emotional expressions such as frustration and confusion were found to be more frequent amongst the unsuccessful problem solvers.

Lester (1994) contended that there was a general agreement that problem difficulty is not so much a function of various task variables such as content and context variables, structure variables, syntax variables and heuristic behaviour variables (Goldin & McClintok, 1979) as it is of characteristics of the problem solver. Traits such as spatial visualisation, ability to attend to structural features of problems; dispositions such as beliefs and attitudes; and experiential background such as instructional history, and familiarity with types of problems, seemed to Lester to be a function of problem difficulty.

Cognitive and Affective Factors that Contribute to Students' Difficulties in Mathematical Problem Solving

Meyer (1978) noted that although certain prerequisite mathematical concepts and skills are related to problem-solving success, a knowledge of these concepts and skills is not sufficient for successful problem solving. Mayer (1982) suggested that some of the types of knowledge that may be relevant for a psychological basis for understanding mathematical problem solving are:

a) linguistic and factual knowledge - concerning how to encode sentences,

b) schema knowledge - concerning relations among problem types,

c) algorithmic knowledge - concerning how to perform well-defined procedures, and

d) strategic knowledge - concerning how to approach problems.

Lester (1982) postulated that successful problem solving in mathematics is a function of at least five components:

a) mathematical knowledge and experience,

b) skill in the use of a variety of generic "tool" skills (e.g., sorting relevant from irrelevant information, drawing diagrams, etc.),

c) the ability to use a variety of heuristics known to be useful in mathematical problem solving,

d) knowledge about one's own cognitions before, during, and after a problem-solving episode, and

e) the ability to maintain executive control (i.e. to monitor and regulate) of the procedures being employed during problem solving.

Schoenfeld (1983a, 1983b) distinguished three types of knowledge needed in problem solving:

a) resources - typically, domain-specific knowledge such as facts and algorithms, routine procedures and heuristics, representations, and other knowledge possessed by the individual which can be brought to bear on the problem at hand,

b) control - planning, monitoring, assessment, "metacognitive" acts and other ingredients related to the selection and implementation of tactical resources, and

c) belief systems - about self, the environment, the topic, and mathematics that influence an individual's behaviour.

Heller and Hungate (1985) summarised the nature of the knowledge required for solving problems in complex subject-matter domains as:

a) knowledge for understanding and representing problems,

b) strategic knowledge which governs the approach problem solvers take to the task,

c) knowledge of basic concepts and principles, and

d) repertoires of familiar patterns and known procedures.

Kilpatrick (1985a) in his retrospective account of the past twenty-five years of research on teaching mathematical problem solving, stated that studies of expert problem solvers and computer simulation models have shown that the solution of a complex problem requires:

a) a rich store of organised knowledge about the content domain,

b) a set of procedures for representing and transforming the problem, and

c) a control system to guide the selection of knowledge and procedures.

Groves and Stacey (1988) claimed that successful problem solving depends on many things - emotional aspects such as being able to control panic, good habits such as not erasing work which may be needed later, as well as awareness and facility with mathematical processes such as generalising, looking at special cases and making conjectures.

In their study of the problem-solving research literature, Kroll and Miller (1993) identified three major cognitive and affective factors, namely knowledge, control (metacognition) and beliefs and affects that contributed to students' difficulties in problem solving.

1. <u>Knowledge Factors</u>

From the literature it appears that students must possess relevant knowledge and be able to coordinate their use of appropriate skills in order to solve problems efficiently. According to Kroll and Miller (1993) there are five kinds of knowledge factors:

1. *Algorithmic Knowledge*

Computational skills and their mastery are a necessary but not sufficient component of problem solving. Students who cannot carry out basic computations will have difficulty solving problems. However, surveys like the British Assessment of Performance Unit (APU) (Eggleston, 1983) and the National Assessment of Educational Progress (NAEP) (Carpenter et al., 1980) show that the percentage of children able to solve a problem requiring a particular computation is often strikingly lower than the percentage of children who can perform that same computation correctly when not part of a problem. Kouba et al. (1988) and Dossey et. al. (1989) also reported that many students could calculate accurately in computation skill tests but were unable to apply these same skills to anything other than the most routine of mathematical problem-solving situations.

2. *Linguistic Knowledge*

Difficulty with the language in which the problem is presented has been proposed as an obstacle to students' successful mathematical problem solving. Davidson (1977) found that low verbal ability or lack of familiarity with the language used in problems did hamper children's understanding of word problems. Although some researchers have claimed that reading ability is a key factor in problem-solving success (Ballew & Cunningham, 1982), others have maintained that reading plays a minor role, especially when students are familiar with the words used in the problems (Balow, 1964; Knifong & Holtan, 1977). It appears that it may be an oversimplification to directly equate reading difficulties with problem-solving difficulties. Research by Muth (1984) and Muth and Glynn (1985) has investigated how reading and computation skills work together in successful problem solving and concluded that reading ability and computational ability both play important roles in children's successful solution of word problems.

3. *Conceptual Knowledge*

Zweng (1979) argued that it was neither computation skills nor reading skills that were the major obstacles when students had difficulty solving problems; rather, it was deciding which mathematical operation to perform. Sowder (1988) found that students were able to compute with whole numbers and to solve single-step word problems but were unable to decide which operations to use when faced with more challenging problems. Lester (1985) also found from his work with third, fourth and fifth graders that their choice of operation was determined by the key words in the problem, usually found in the last sentence or question. Such a situation arises when students are taught to rely on key words (Nesher & Teubal, 1975) to decide which operations to use rather than their knowledge of concepts which are embodied in the problems.

4. *Schematic Knowledge*

Good problem solvers are more likely than poor problem solvers to remember information about the structure of the problems they have solved previously, to learn from their mistakes and to relate work they have done on previous problems to the task at hand (Silver, 1981). Krutetskii (1976) found that good problem solvers possessed the ability to see the mathematical structure of a problem quickly and accurately and to remember its formal structure for a considerable period of time. Schoenfeld and Hermann (1982) found that novices attended to surface features of problems, whereas experts categorised problems on the basis of the fundamental principles involved. Span and Overtoom-Corsmit (1986) found that good problem solvers took more time to analyse the situation to produce an appropriate schema and clear representation of the problem.

5. *Strategic Knowledge*

Problem solving requires knowledge of strategies. Students need techniques that will help them develop plans for a solution (Polya, 1945). Strategic knowledge governs the approach problem solvers take to the task. Heller and Hungate (1985) found that experts perform high-level planning and qualitative analysis before beginning to generate equations, while novices tend to go directly from the problem text to equations. Span and Overtoom-Corsmit (1986) found that good problem solvers tried to schematise to work systematically with more than one strategy. Good problem solvers also often have a wide range of strategies at their disposal and demonstrate facility in choosing which of these strategies to apply in various situations (Kroll & Miller, 1993).

2. <u>Control Factors</u>

Schoenfeld (1981) has reported instances in problem-solving protocols of college students where the problem solvers made a disastrous decision at the planning stage of a problem resulting in no progress towards a solution. These students were very knowledgeable about the mathematical content and were instructed in problem-solving strategies, but they made a "bad" decision.

Lester (1985) stated that in addition to having sufficient domain-general and domain-specific knowledge, successful problem solving also depends upon knowing when and how to utilise such knowledge and upon having the ability to monitor and evaluate the application of this knowledge, both during and after implementation.

The term "metacognition" was first used by Flavell (1976) to refer to "one's knowledge concerning one's own cognitive processes and products or anything related to them" and generally refers to the ability to control one's own thinking processes in problem solving. Metacognition has been suggested as the "driving force" in children's mathematical problem solving (Silver, 1982a) and is important to success in problem solving, in which the solver must make decisions about which strategy to apply and how long to keep on trying it before stopping and selecting a new strategy (Silver, 1982b; Schoenfeld, 1983a). According to Schoenfeld:

> *One of the hallmarks of good problem solvers' control behaviour is that, while they are in the midst of working problems, such individuals seem to maintain an internal dialogue regarding the way that their solutions evolve. (1985, p. 140)*

Kilpatrick (1985b) also considered that problem solvers' ability to reflect on their own cognitive processes would bring to consciousness an awareness of their emotional reactions to problem solving. An increased awareness of these emotional influences should give problem solvers greater control over their cognitive processes.

Siemon (1986), in her study on how reflection can transform teaching and learning in the mathematics classroom, found that reflection amongst problem solvers promotes understanding, provides motivation and increases confidence thereby leading to improved problem-solving performance and more efficient learning. In Foong's (1990) study of successful and non-successful problem-solving processes used by pre-service teachers from Singapore, the successful problem solvers' metacognitive behaviours were task-directed, exhibiting a situational awareness of how things were and where they should be going in the process. On the other hand, the unsuccessful solvers were more aware of their own uncertainties and confusion rather than focusing on the process itself; they tended to be stuck in a certain line of attack without realising it or they would go on a "wild goose chase". Taplin (1994) found that the "perseverers" exhibited more control over their actions during problem solving and were also more inclined than the "non-perseverers" to be flexible in their use of strategies.

Stacey (1990), in her investigation of students' capacities to use their mathematical knowledge in relatively unfamiliar problem-solving situations, found that the below average (D) level student, as rated by the class teacher, outperformed the above average (A) student because of a well developed intuitive strategy of "guess and check" which coupled with a positive attitude and self-reliant ways to monitor progress, allowed the student to make optimal use of a somewhat limited mathematical knowledge.

3. Beliefs and Affective Factors

When students approach mathematical tasks, and especially those that are problematic, they do not enter the arena as purely cognitive beings. Students' behaviours may be influenced by their feelings of self-esteem, their perceived control - or lack of control - over the situation with which they are faced, or their sense of satisfaction in engaging in mathematical tasks (Silver, 1985). Beliefs and affective factors can assist or interfere with problem solving. After a year of observing over 700 intermediate grade children solving problems, the staff of the Mathematical Problem Solving Project (MPSP) decided that willingness, perseverance, and self-confidence were three of the most important influences on problem-solving performance (Webb, Moses, & Kerr, 1977).

Silver (1982a) speculated that affective factors like confidence and willingness to persist may have a substantial effect on the metacognitive processes of problem solvers. Lester, Garofalo, and Kroll (1989), from their studies on students' attitudes and beliefs, conjectured that an individual's beliefs about self, mathematics, and problem solving play a dominant, often overpowering, role in his or her problem-solving behaviour. Schoenfeld (1983a) also emphasised the role that belief systems play in determining the kinds of managerial decisions that problem solvers make. He suggested that attitudes toward mathematics and confidence about mathematics may be aspects of student belief systems that have an important effect on how students manage their cognitive resources. Schoenfeld (1985) and Silver (1985) pointed out that students' beliefs about mathematics may weaken their ability to solve nonroutine problems. If students believe that mathematical problems should always be completed in five minutes or less, they may be unwilling to persist in trying to solve problems that may take substantially longer. Students' confidence in themselves as problem solvers, or their beliefs and feelings about the nature of mathematics - its inherent structure or lack of structure - can exert a strong prohibitive force on their ability to solve or even initially attack problems in a productive way (Shaughnessy, 1985).

Lester and Garofalo (1982) reported that third and fifth graders believed that mathematical problems could always be solved by using basic operations and could always be solved in only a few minutes. Students' responses to attitude items on the Third National Assessment of Educational Progress in Mathematics (Carpenter et al., 1983) indicated the general belief among junior high and high school students that there was always one correct way to solve any mathematical problem, and that mathematics is mostly memorisation. Research by Lesh (1983) suggested that many students do not believe that mathematics is applicable to solving real-world problems.

Trimmer (1974) found that confidence, lack of anxiety, flexibility, lack of rigidity, and an ability to cope with uncertainty were traits associated with successful problem solving. Foong (1990) noted that when unsuccessful problem solvers failed to obtain results, they were easily irritated and their subsequent behaviours became self-directed and task-irrelevant. They expressed a lack of self confidence and would continue in a rambling manner or they gave up, thus avoiding further frustration.

Concluding Remarks

Problem solving in mathematics can be described as "thinking and working mathematically" but the converse is not true. Problem solving in mathematics is a complex process which requires an individual who is engaged in a mathematical task to coordinate and manage domain-specific and domain-general pieces of knowledge.

Successful and unsuccessful problem solvers in mathematics differ with regard to knowledge, control and beliefs and affect factors. The review of the research literature on factors that contribute to students' difficulty in mathematical problem solving suggests that:

1. the mathematics content level of the problems which students at different year levels of schooling will be able to solve successfully, and

2. the different strategies or heuristics which students at different year levels use to solve the same mathematical problems

must govern the design of problem-solving curricula at the various year levels of schooling.

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